Fast-responding nonlinear phase shifter using a signal-wave gain medium

K. J. Jiang,[1,2] L. Deng,[1] E. W. Hagley,[1] and M. G. Payne[3]

[1]*Electron & Optical Physics Division, National Institute of Standards & Technology, Gaithersburg, Maryland 20899, USA*

[2]*Center for Cold Atom Physics, Wuhan Institute of Physics and Mathematics, CAS, Wuhan 200062, China*

[3]*Department of Physics, Bowling Green State University, Bowling Green, Ohio 43403, USA*

(Received 30 October 2007; published 22 April 2008)

Using a full density matrix formalism we show that for a lifetime broadened four-level scheme with a signal wave gain medium a large nonlinear phase shift can be induced without signal wave slowdown and attenuation. In this system the signal wave acquires a large nonlinear phase shift and travels with *superluminal* propagation characteristics. This raises the possibility of rapidly responding nonlinear phase shifting and possibly phase gating devices for information science.

DOI: 10.1103/PhysRevA.77.045804 PACS number(s): 42.65.Dr, 42.50.Gy

Control of the slowly varying phase of an electromagnetic wave has great technological importance in the field of information science. In solid state media such as optical fibers, nonlinear phase shift of a signal wave can be induced by using self- or cross-phase modulation (SPM or CPM) [1] effects. These effects, however, usually are very weak, requiring an extended propagation distance to establish. In gaseous phase media, distinctive energy levels and transition selection rules imply that with sufficiently small detunings nonlinear effects such as SPM and CPM can be strongly enhanced. A typical four-level scheme [2] using continuous wave (CW) and weakly driven electromagnetically induced transparency [3] (EIT) technique is based on these considerations. This steady-state nonlinear phase shift scheme achieves the enhancement of the Kerr effect by using a weakly on-resonance control field to significantly reduce the propagation velocity of a signal carrier field (often called a probe field). Such an N scheme and its variations have since been used in theoretical studies of quantum entanglement of ultraslow photons [4], single photon switching [5], nonlinear phase gates [6], and single photon propagation controls [7].

In this Brief Report we propose a scheme for significant enhancement of nonlinear phase shift of a signal field. The scheme is based on an active-Raman-gain (ARG) configuration recently demonstrated [8,9] to be able to produce both superluminal and ultraslow propagation of optical waves. The key idea of the ARG scheme is that the signal field operates in a stimulated Raman *emission* mode. This is very different from all EIT-based schemes where the signal field operates in an *absorption* mode. It is precisely this emission mode that gives a signal wave novel propagation characteristics and properties [8–10]. Indeed, we show that a signal wave can acquire a large nonlinear phase shift, suffer no attenuation and distortion (the signal actually has a small gain), yet travels with a *superluminal* group velocity (therefore rapid device transient respond time). The latter two features are unattainable with any weakly driven EIT-based scheme where ultraslow propagation, significant signal wave attenuation and distortion, and long response time are inherently unavoidable for the signal or probe wave.

We consider a lifetime broadened four-level system [Fig. 1(a)] where a strong CW pump laser (Rabi frequency $2\Omega_P$) couples the ground state $|1\rangle$ to an excited state $|2\rangle$ with a large one-photon detuning δ_2. A weak pulsed (pulse length τ) signal laser [Rabi frequency $2\Omega_s(z,t)$ with slowly varying amplitude $2\Omega_s^{(0)}(z,t)$] couples state $|2\rangle$ to a lower excited state $|3\rangle$ with a larger one-photon detuning $\delta_3 \approx \delta_2$, resulting in a two-photon detuning $\delta_{2p}=\delta_3-\delta_2$ for the two-photon transition $|1\rangle-|3\rangle$. A CW laser (Rabi frequency $2\Omega_4$) couples state $|3\rangle$ to a different excited state $|4\rangle$ which does not have a dipole-allowed transition back to the ground state $|1\rangle$. As a comparison the commonly used EIT-based scheme is given in Fig. 1(b), where the transitions coupled by the pump and signal carrier fields are interchanged.

We use perturbation theory to seek the nonlinear response of the system. The first step is to work out the system response in the absence of the phase-control field E_4. This solution is then used as the zeroth order solution to seek the nonlinear perturbation correction to the signal field due to the presence of the phase-control field E_4 [11]. The density matrix equations of motion describing the ARG scheme [Fig. 1(a)] are given by

$$\dot{\rho}_{31} = i\delta_{2p}\rho_{31} + i\Omega_{32}\rho_{21} - i\Omega_{21}\rho_{32} - \gamma_{31}\rho_{31}, \tag{1a}$$

$$\dot{\rho}_{32} = -i\delta_3\rho_{32} - i\Omega_{12}\rho_{31} - \gamma_{32}\rho_{32}, \tag{1b}$$

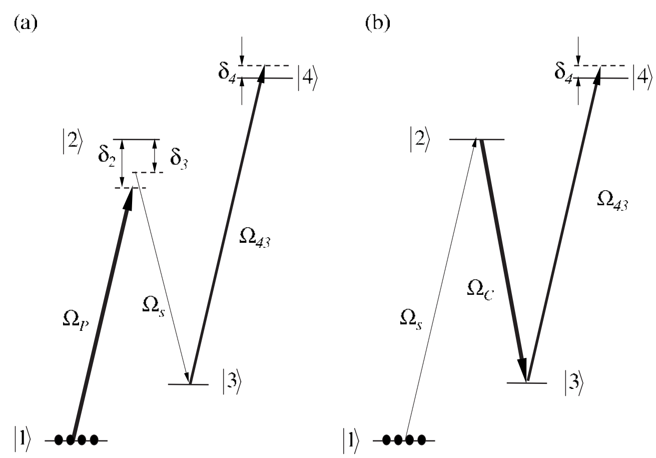

FIG. 1. (a) The proposed ARG-core based schemes. $\Omega_P=\Omega_{21}$, $\Omega_s=\Omega_{23}$. Possible energy levels using ^{85}Rb: $|1\rangle=|F=2,m_F=-2\rangle$, $|2\rangle=|F'=3,m_{F'}=-1\rangle$, and $|3\rangle=|F''=3,m_{F''}=0\rangle$, $|4\rangle=|F'''=4\rangle$. (b) The commonly used EIT-based scheme. $\Omega_C=\Omega_{23}$, $\Omega_s=\Omega_{21}$.

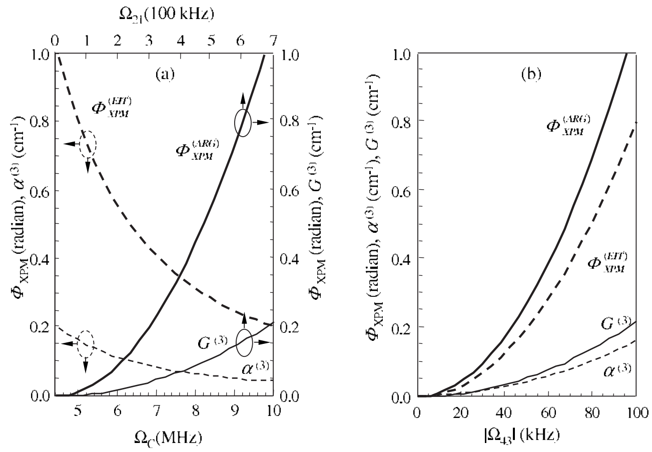

FIG. 2. (a) Nonlinear phase shifts, attenuation coefficient for an EIT-based scheme (thick and thin dashed lines, $\alpha^{(3)}$), and gain coefficient for an ARG-based scheme (thick and thin solid lines, $G^{(3)}$). The nonlinear phase shift field Rabi frequency $|\Omega_{43}=100$ kHz for both cases. (b) Nonlinear phase shifts, attenuation, and gain coefficients as functions of the phase-shift laser field Rabi frequency $|\Omega_{43}|$ for EIT-based and ARG-based schemes.

$$\dot{\rho}_{21} = i\delta_2\rho_{21} + i\Omega_{21}\rho_{11} + i\Omega_{23}\rho_{31} - \gamma_{21}\rho_{21}. \tag{1c}$$

Here, $2\Omega_{jk}=D_{jk}E_L/\hbar$ and γ_{jk} are the Rabi frequency and decoherence rate for the $|j\rangle-|k\rangle$ transition with dipole moment D_{jk} under the appropriate excitation with the laser field E_L $(L=s,P)$. Note that since $|\delta_2\tau|\gg 1$, $|\delta_3\tau|\gg 1$, and the probe laser is weak, the ground state population is not depleted and Eq. (1c) can be evaluated adiabatically, yielding

$$\rho_{21} \approx -\frac{\Omega_{21}\rho_{11}}{(\delta_2 + i\gamma_{21})}. \tag{2}$$

Substituting Eq. (2) into Eq. (1a) and neglecting higher-order terms, Eqs. (1a) and (1b) can be solved analytically using a standard time Fourier transform method. Under the low gain condition we have, in Fourier space,

$$\alpha_{31} \approx \frac{d_{32}\Omega_{21}}{D_0}\Lambda_{32}, \quad \alpha_{32} \approx \frac{|\Omega_{21}|^2}{D_0}\Lambda_{32}, \tag{3}$$

where $D_0=(\delta_2+i\gamma_{21})d_{31}d_{32}$, $d_{31}=\omega+\delta_{2p}+i\gamma_{31}$, $d_{32}=\omega-\delta_3+i\gamma_{32}$, and $|\Omega_{21}|^2\ll|d_{31}d_{32}|$. In addition, α_{jk} and Λ_{jk} are the time Fourier transforms of ρ_{kj} and Ω_{jk}, respectively, and ω is the transform variable.

When the phase control field E_4 is added, the following two equations of motion must be considered:

$$\dot{\rho}_{41} = i\delta_{3p}\rho_{41} + i\Omega_{43}\rho_{31} - i\Omega_{21}\rho_{42} - \gamma_{41}\rho_{41}, \tag{4a}$$

$$\dot{\rho}_{42} = i\Delta_{2p}\rho_{42} + i\Omega_{43}\rho_{32} - i\Omega_{12}\rho_{41} - \gamma_{42}\rho_{42}. \tag{4b}$$

Here, $2\Omega_{43}=D_{43}E_4/\hbar$, the three-photon detuning for $|1\rangle-|4\rangle$ pumping is $\delta_{3p}=\delta_4-\delta_{2p}$ whereas the two-photon detuning for transition $|2\rangle-|4\rangle$ is $\Delta_{2p}=\delta_3-\delta_4$. Taking $|\Omega_{21}|^2\ll|d_{41}d_{42}|$, where $d_{41}=\omega+\delta_{3p}+i\gamma_{41}$ and $d_{42}=\omega-\Delta_{2p}+i\gamma_{42}$, and using α_{31} and α_{32} given in Eq. (3), the nonlinear correction to the density matrix element can be written as

$$\rho_{32}^{(NL)} = C_0\int_{-\infty}^{+\infty}\frac{d\omega\Lambda_{32}e^{-i\omega t}}{(\omega+\delta_{2p}+i\gamma_{31})^2}, \tag{5}$$

where

$$C_0 \approx -\frac{|\Omega_{43}|^2|\Omega_{21}|^2}{(\delta_2+i\gamma_{21})(\delta_3-i\gamma_{32})(\delta_{3p}+i\gamma_{41})}. \tag{6}$$

In deriving Eq. (5) we assumed that for low gain conditions δ_2 and δ_3 are much larger than any Rabi frequencies, multiphoton detunings, and frequency shifts induced by both pump and nonlinear-phase-control laser fields. In addition, we assumed that $|\delta_{3p}\tau|\gg 1$ [12].

The inverse transform Eq. (5) depends on the Fourier transform variable ω. This is also the case of the weakly driven EIT-based scheme when operated in the short-pulsed regime. In an ARG scheme, this difficulty can be easily circumvented by assuming $|\delta_{2p}\tau|>5$ [13]. This condition leads to a fairly accurate inverse transform for an input signal field with a Gaussian pulse envelope [14]

Assuming $|\delta_{2p}\tau|>5$ and taking $\Omega_s(0,t)=\Omega_s^{(0)}(0,0)e^{-t^2\tau^2}$ so that $\Lambda_{32}(0,t)=(\tau/\sqrt{2})\Omega_s^{(0)}(0,0)e^{-\omega^2\tau^2/4}$ where $2\Omega_s^{(0)}(0,0)$ is the Rabi frequency of the signal field at the entrance of the medium, Eq. (5) can be evaluated analytically with good accuracy. The nonlinear polarization at the signal wave frequency can now be constructed to give

$$\text{nonlinear source term} = (i\Phi_{XPM}+G^{(3)})E_s,n \tag{7a}$$

where we have defined the cross-phase modulation phase shift per unit length and the third-order nonlinear gain coefficient at the signal wave frequency as

$$\Phi_{XPM}(\omega_s) = -\kappa_{23}\frac{|\Omega_{21}|^2}{\delta_2\delta_3\delta_{2p}^2}\left(\frac{|\Omega_{43}|^2\delta_4}{\delta_4^2+\gamma_{41}^2}\right), \tag{7b}$$

$$G^{(3)}(\omega_s) = \kappa_{23}\frac{|\Omega_{21}|^2}{\delta_2\delta_3\delta_{2p}^2}\left(\frac{|\Omega_{43}|^2\gamma_{41}}{\delta_4^2+\gamma_{41}^2}\right). \tag{7c}$$

Here, $\kappa_{23}=2\pi\omega_s N D_{23}/(\hbar c)$, and we have neglected γ_{21},γ_{32} in comparison with δ_2 and δ_3, and taken $|\delta_{2p}|\ll|\delta_4|$ so that $\delta_{3p}\approx\delta_4$. We note that Eq. (7c) leads to a third-order nonlinear *gain* rather than the usual third-order *attenuation* as encountered in weakly driven EIT-based schemes [2]. This is precisely the consequence of the signal wave being operated in a *stimulated emission mode*.

We emphasize that the new scheme described here does *not* rely on ultraslow propagation of the signal wave. Indeed, using Eq. (3) and Maxwell's equation for the signal wave it is straightforward to show that the signal wave travels *superluminally* [9,10]. This key feature eliminates the necessity of a two-species medium and the requirement of matching the group velocities of an ultra slow signal wave and the phase-control field [4], and may leads to rapidly responding nonlinear phase switching and phase gates for information science. For example, consider a device with a medium length of $z=1$ cm. If the signal wave group velocity is reduced to $V_g\approx 10^4c$ using the weakly driven EIT scheme, then it will take the signal wave about 300 ns to pass the device during which the nonlinear phase-shifting field Ω_{43} must be present all the time. On the other hand, for the signal wave propagation in an ARG system the device transient time is only about 30 ps [15] to acquire the similar nonlinear phase shift.

In Fig. 2 we compare an ARG-based scheme with a typical EIT-based scheme for nonlinear phase control using experimentally feasible parameters for cold atomic vapors.

Weakly driven EIT-based scheme: $\kappa_{12} \approx 10^{11}/(\mathrm{m}\cdot\mathrm{s})$, $\gamma_{31} \approx 100$ Hz, $\gamma_{21} \approx 5.7$ MHz, $\gamma_{41} \approx 100$ kHz, and $\delta_2 = \delta_3 = \delta_{2p} = 0$. For a signal wave pulse length of $\tau = 50$ μs, we take $\delta_4 = 5\gamma_{41}$ and $|\Omega_{43}| = 100$ kHz, and third-order field attenuation coefficient $\alpha^{(3)} \approx 0.2$ cm^{-1} (thin dashed line) for $\Phi^{(EIT)}_{XPM)} \approx 1$ radian (thick dashed line). This implies that the signal wave intensity attenuation will be $e^{-0.4}$ (about 33%) for a 1-cm device.

ARG-based scheme: $\kappa_{32} \approx 10^{11}/(\mathrm{m\ s})$. In addition to the parameters given above, we choose $\delta_2 = 30$ MHz, $\delta_3 = 2.99$ MHz, $\delta_{2p} = -100$ kHz, and $|\Omega_{43} = 100$ kHz. With these parameters, we have $|\delta_{2p}\tau| = 5$, $|\delta_4\tau| = 25 \gg 1$, and all conditions required are satisfied and our analytical results are valid. The third-order gain $G^{(3)} \approx 0.2$ cm^{-1} (thin solid line) for $\Phi^{(ARG)}_{XPM} \approx 1$ radian (thick solid line) [16]. Clearly, the ARG-based scheme works better than the EIT-based scheme.

In Fig. 2(b) we plot the signal wave nonlinear phase shift, absorption (EIT) and gain (ARG) coefficients as functions of the nonlinear-phase-inducing-laser Rabi frequency $|\Omega_{43}|$. We have chosen $|\Omega_{21}| = 70$ kHz and $|\Omega_C| = 5$ MHz (note that further reducing Ω_C significantly attenuates and distorts the signal wave). All other parameters are the same as in Fig. 2(a). Again, the ARG-based scheme works better than the EIT-based scheme. We note that it is always possible to make that $|V_g^{(ARG)}| = |V_g^{(EIT)}|$. This leads to $|\Phi^{(ARG)}_{XPM}| = |\Phi^{(EIT)}_{XPM}|$ and $|G^{(3)}| = |\alpha^{(3)}|$.

In summary, we have presented a four-level lifetime broadened N scheme for enhancement of the nonlinear phase shift of a signal wave. Contrary to widely used weakly driven EIT-based schemes the scheme studied here is based on the principle of operating the signal wave in a stimulated Raman emission mode. The gain-assisted nature of this emission process leads to interesting features such as large nonlinear phase shifts with nondistorted gain and superluminal propagation velocities. It eliminates the necessity of multispecies medium and the requirement of matching the ultraslow group velocities of the signal wave and the nonlinear-phase control field. It may also lead to the possibility of fast nonlinear phase switching and nonlinear phase gates for information science. Finally, we note that the treatment can be generalized to describe a quantum field at the single photon level. A full quantum theory of this scheme and its application in phase shift, nonlinear gates, and the entanglement of photons will be reported elsewhere.

[1] G. P. Agrawal, *Nonlinear Fiber Optics* (Elsevier Scientific, Amsterdam, 2001).

[2] H. Schmidt and A. Imamoglu, Opt. Lett. **21**, 1936 (1996).

[3] A. Kasapi, M. Jain, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **74**, 2447 (1995).

[4] M. D. Lukin and A. Imamoglu, Phys. Rev. Lett. **84**, 1419 (2000).

[5] S. E. Harris and Y. Yamamoto, Phys. Rev. Lett. **81**, 3611 (1998).

[6] C. Ottaviani, D. Vitali, M. Artoni, F. Cataliotti, and P. Tombesi, Phys. Rev. Lett. **90**, 197902 (2003); D. Petrosyan and G. Kurizki, Phys. Rev. A **65**, 033833 (2002); I. Friedler, G. Kurizki, and D. Petrosyan, *ibid.* **71**, 023803 (2005).

[7] M. Lukin and A. Imamoglu, Nature (London) **413**, 273 (2001).

[8] K. J. Jiang, L. Deng, and M. G. Payne, Phys. Rev. A **74**, 041803(R) (2006).

[9] K. J. Jiang, L. Deng, and M. G. Payne, Phys. Rev. A **76**, 033819 (2007).

[10] M. G. Payne and L. Deng, Phys. Rev. A **64**, 031802(R) (2001), and references on superluminal propagation and gain-assisted schemes therein.

[11] We have neglected corrections due to a very weak spontaneous Raman emission near the frequency of E_s.

[12] The dominant contribution to δ_{3p} is $|\delta_4| \gg |\delta_{2p}|$.

[13] In EIT-based schemes $|\delta_{2p}\tau| > 1$ leads to severe penalties.

[14] Without this condition it is in general incorrect to make a steady-state approximation, especially if $\gamma_{31}\tau < 1$. This conclusion applies equally to all weakly-driven EIT-based schemes. See M. Payne, L. Deng, and K. Jiang, Phys. Rev. A **74**, 043810 (2006).

[15] Typically, the lead time is about 10% of c. Thus it is quite accurate to estimate the device transient time using free-space speed of light.

[16] There is a gain-induced static [i.e., Ω_P dependent, see Fig. 1(a)] phase. This static phase is independent of the nonlinear phase shift laser field Rabi frequency Ω_{43} and can be effectively eliminated technically.